Exercise III

1. Sketch the graph of each of the following functions:
   (i) $y = 10^x$, (ii) $y = 2^x$, (iii) $y = 3^{-x}$, (iv) $y = 3^{-x^2}$
   (v) $y = e^x$, (vi) $y = e^{-x^2}$, (vii) $y = \log_{10}(x)$,
   (viii) $y = \log_2(x)$, (ix) $y = \log_e(x)$.

2. Determine whether each of the functions in question 1 is bounded above, bounded below, bounded respectively.

3. Sketch the graph of a function f with domain [1,5], $f(1) \neq f(5)$, which does not take on all values between $f(1)$ and $f(5)$

4. Find the solution sets for each of the following inequalities:
   (i) $|x - 3| + 7 < 12$; (ii) $|4x + 24| < 12$; (iii) $2|x + 4| = 16$.

5. (i) Use absolute value notation to express the fact that a real number x is less than a distance 0.5 from 1 on the number line.
   (ii) Use interval notation to describe the set of real numbers which are less than a distance 0.5 from 1 on the number line.

6. Express each of the following series as an infinite decimal:
   (i) $\sum_{k=1}^{\infty} \frac{1}{10^{\frac{k(k+1)}{2}}}$, (ii) $\sum_{k=1}^{\infty} \frac{3}{10^{k^2}}$.

7. For each of the following find a number T such that $|a_n - L| < \epsilon$ for all $n > T$.

   (i) $a_n = \dfrac{n+1}{n}, L = 1, \epsilon = \frac{1}{6}$;

   (ii) $a_n = \dfrac{2n}{3n+2}, L = \frac{2}{3}, \epsilon = \frac{1}{24}$;

   (iii) $a_n = \dfrac{(-1)^n}{n}, L = 0, \epsilon = 0.2$.

   Sketch a suitably annotated graph which will illustrate each of the above.
   The first one is done for you below: